Engineering Notes

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Inviscid Interaction of Trailing Vortex Sheets Approximated by Point Vortices

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Nomenclature

Æ	= aspect ratio of a wing
b	= wing span
c	= wing chord
d_0	= initial separation between the centroids of two vortex sheets
N	= number of vortices
t	= time
U_∞	= freestream velocity, aligned with x-axis
u,v,w	= velocity components in x,y,z directions, respectively
x_m	= merging distance downstream of the wing
x,y,z	= Cartesian coordinate system, x - streamwise, z - vertical
α	= angle of attack
Γ_0	= circulation at wing root
γ	= circulation in point vortices

Introduction

INCREASED use of wide-body jet transport aircraft in this decade has renewed interest in devising means to alleviate the hazard to following aircraft caused by the high-rotational velocities in the wake of the generating aircraft. NASA wind-tunnel[1] and flight tests[2] suggest that the wing lift distribution may be altered such that two (or more) corotational vortices are shed from each half of the wing. The merging of these vortices leads to a single vortex with a sufficiently diffused vorticity distribution, consequently reducing the harzard.

The phenomenon of merging itself is only partially understood. Recent investigations[3-6] of a theoretical and experimental nature have contributed significantly toward answering many questions, along with raising some unanswered ones. In Ref. 6, the results of an experimental study of the merging of two corotational vortices of equal strength were presented. The data were acquired in an open-circuit wind tunnel using half-span wing models inserted through adjacent or opposite sides of the test section to generate interacting vortices. The variation of the nondimensional merging distance with initial vortex spacing for various model configurations was found to be approximately linear and expressed as

$$x_m\Gamma_0/U_\infty bc = 6.69\ (d_0/d_\Gamma) - 4.47 \tag{1}$$

where the nondimensionalizing parameter, d_Γ, on the right-hand side is the diameter of the Rankine vortex having the same circulation defect as that of the given vortex.[6] Equation (1) shows that the merging distance goes to zero for

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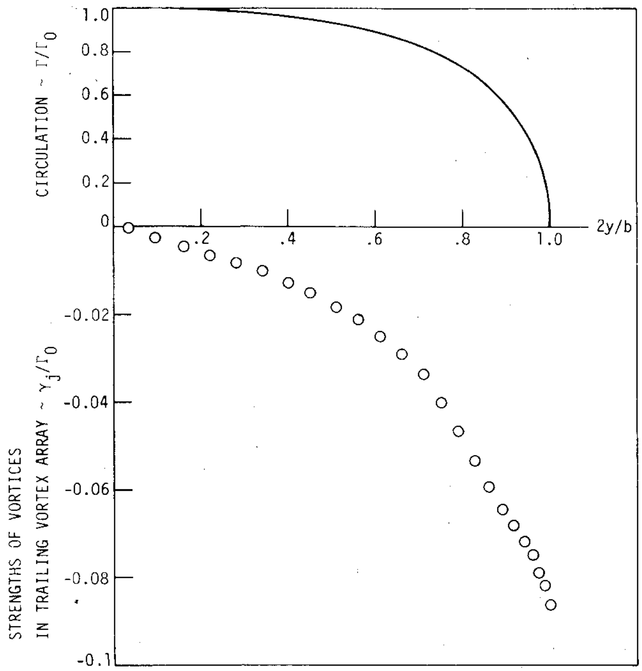

Fig. 1 Circulation distribution on a) a semispan NACA 0012, elliptical-tip wing, Æ = 5.0, α = 11 deg, and b) the trailing vortex sheet approximated by an array of 25 vortices.

$d_0/d_\Gamma = 0.668$. It was concluded that for initial spacing less than 0.668 the vortices roll-up already merged. The latter was also observed for short but nonzero merging distances. This conclusion is corroborated by the results of the present computational study of inviscid roll-up of two trailing vorticity sheets.

Mathematical Model

Glauert's lifting line theory is used to obtain spanwise circulation distribution on the vortex generator wing used in Ref. 6, which is a semispan, symmetrical cross-section, constant chord wing, except near the tip where the chord distribution is elliptical. Any modification of the spanwise ciruclation distribution of the wing due to the presence of the other wing in the vicinity has been ignored. The trailing vortex sheet of each wing is approximated by an array of 25 unequally spaced vortices whose strengths are given by $\gamma_j = \Delta\Gamma_j$, as shown in Fig. 1. The subsequent motion of the vortices is computed in a Trefftz plane. The implicit asumption is that the streamwise gradients are small so that the motion can be treated as an unsteady two-dimensional motion. The two components of velocity for any point vortex are given by

$$v_i = \frac{\Delta y_i}{\Delta t} = -\sum_{j\neq i}^{N} \frac{\gamma_j (z_i - z_j)}{2\pi[(y_i - y_j)^2 + (z_i - z_j)^2]} \tag{2}$$

$$w_i = \frac{\Delta z_i}{\Delta t} = \sum_{j\neq i}^{N} \frac{\gamma_j (y_i - y_j)}{2\pi[(y_i y_j)^2 + (z_i - z_j)^2]} \tag{3}$$

The locations of the N vortices at an advanced time are found by using velocity over the time increment Δt.[7] The computer program in Ref. 5 is applied to track the trailing vortices. The program monitors changes in the Kirchhoff-Routh path function and the first and second moments of circulation for each of the vortex sheets, thereby preventing the numerical results from being meaningless.

Results

The mutual interaction of the trailing vortex sheets for an initial spacing $d_0/d_\Gamma = 0.77$ is shown in Fig. 2. The configuration is in accordance with the experimental setup. The results clearly show that the two sheets roll up together and eventually approach a nearly circular shape. The distance between the centroids of each of the vortex sheets decreases as the distance downstream increases. The case of a larger initial spacing, $d_0/d_\Gamma = 1.09$, is shown in Fig. 3. A strong interaction between the two sheets is observed and they merge to form a single vortex of nearly circular shape. These results are in good qualitative agreement with the experimental observations.[6] In Fig. 4, the case of $d_0/d_\Gamma = 1.66$, close to the value corresponding to the inviscid merging boundary,[7] is shown. The two sheets start rolling up independently and exhibit only a weak interaction further downstream. A word of caution about the interpretation of these results, however, is in order here. These results should be considered to represent the gross aspects and trends of the vortex sheets—the detailed motion itself may not be accurately determined. Many studies[7-10] have led to the conclusion that some of the motions of the vortices may be due to the discrete vortices model and may not correspond to the actual behavior of the vortex sheet being represented. In the present study also, the vortices are found to undergo chaotic motion in the region of roll-up.

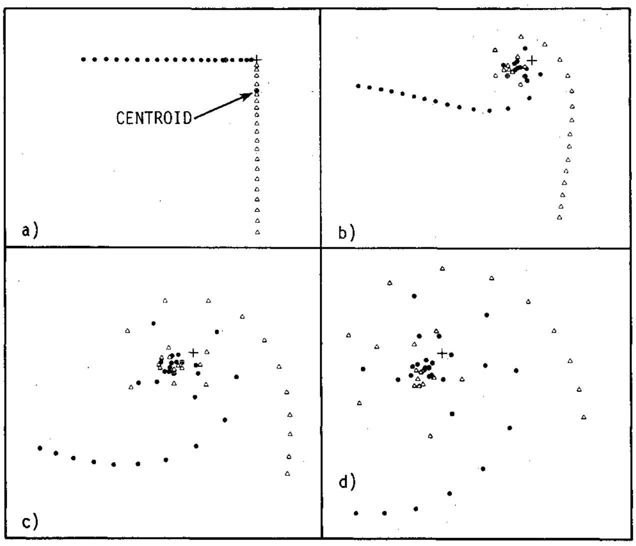

Fig. 2 Inviscid merging of two corotational trailing vortex sheets shed by semispan, NACA 0012, elliptical-tip wings, $\mathcal{R} = 5.0$, $\alpha = 11$ deg, $\Gamma_0/U_\infty b = 0.1$, $d_0/d_\Gamma = 0.77$: a) $x/b = 0.0$; b) $x/b = 1.0$ (sheets roll up together); c) $x/b = 4.0$; d) $x/b = 8.0$ (merged vortex).

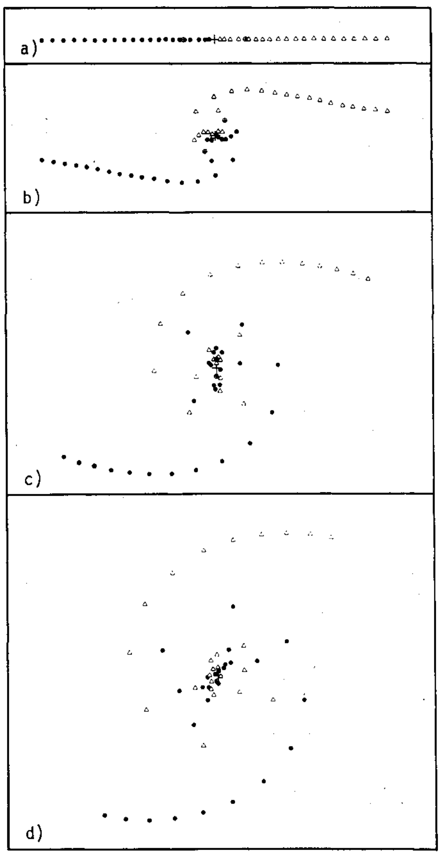

Fig. 3 Inviscid interaction of two corotational trailing vortex sheets shed by semispan, NACA 0012, elliptical-tip wings, $\mathcal{R} = 5.0$, $\alpha = 11$ deg, $\Gamma_0/U_\infty b = 0.1$, $d_0/d_\Gamma = 1.09$: a) $x/b = 0.0$; b) $x/b = 1.0$ (sheets interact strongly during roll-up); c) $x/b = 7.0$ (merged vortex).

Concluding Remarks

A computational study of the mutual interaction of vorticity sheets shed behind wings kept at a small distance apart has been made. The sheets are modeled by arrays of discrete point vortices and their inviscid motion is computed. The error monitors are used in the computations so that plotted results are reliable. The gross features exhibited by the results confirm the experimental observation that for a small initial separation distance, the trailing vorticity sheets roll up together. More accurate prediction of the motion of the sheets would probably require a finite-difference type approach. It should also be noted that the three-dimensional effects, including the influence of a bound vortex, and the viscous effects have not been accounted for.

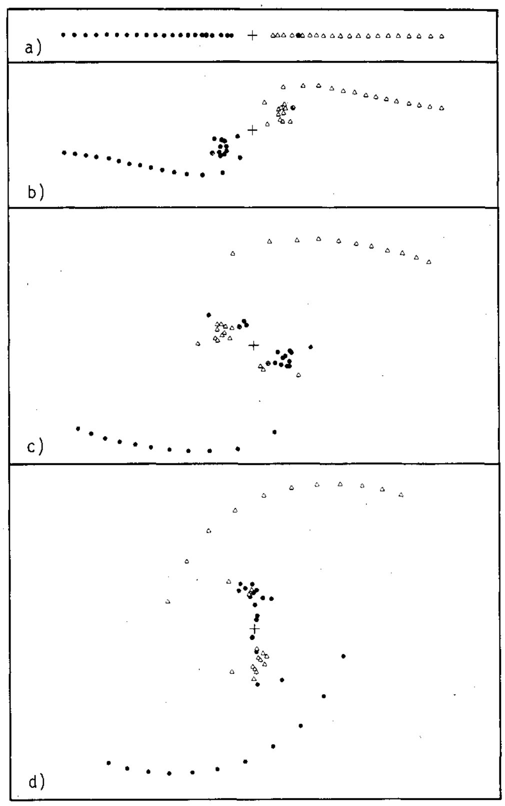

Fig. 4 Inviscid interaction of two corotational trailing vortex sheets shed by semispan, NACA 0012, elliptical-tip wings, $\mathcal{R} = 5.0$, $\alpha = 11$ deg, $\Gamma_0/U_\infty b = 0.1$, $d_0/d_\Gamma = 1.66$; a) $x/b = 0.0$; b) $x/b = 1.0$ (sheets roll up independently); c) $x/b = 4.0$; d) $x/b = 7.0$ (weak interaction).

References

[1] Corsiglia, V. R. and Dunham R. C., Jr., "Aircraft Wake-Vortex Minimization by Use of Flaps," *NASA Symposium on Wake-Vortex Minimization*, Washington, D.C., 1976, pp. 303-336.

[2] Jacobson, R. A. and Barber, M. R., "Flight Test Technique for Wake-Vortex Minimization Studies," *NASA Symposium on Wake-Vortex Minimization*, Washington, D.C., 1976, pp. 191-217.

[3] Bilanin, A. J., Snedeker, R. S., and Teske, M. E., "Interaction and Merging of Line Vortices," AFOSR-TR-76-0873, 1976.

[4] Bilanin, A. J., Teske, M. E., Donaldson, C. du P., and Snedeker, R. S., "Viscous Effects in Aircraft Trailing Vortices," *NASA Symposium on Wake Vortex Minimization*, Washington, D.C., 1976, pp. 55-122.

[5] Rossow, V. J., "Convective Merging of Vortex Cores in Lift-Generated Wakes, *Journal of Aircraft*, Vol. 14, March 1977, pp. 283-290.

[6] Brandt, S. A. and Iversen, J. D., "Merging of Aircraft Trailing Vortices," 15th Aerospace Science Meeting, Los Angeles, Calif., Jan. 24-26, 1977.

[7] Rossow, V. J., "Theoretical Study of Lift-Generated Vortex Wakes Designed to Avoid Rollup," *AIAA Journal*, Vol. 13, April 1975, pp. 476-484.

[8] Moore, D. W., "A Numerical Study of the Roll-Up of a Finite Vortex Sheet," *Journal of Fluid Mechanics*, Vol. 63, Pt. 2, April 1974, pp. 225-235.

[9] Kuwahara, K. and Takami, H., "Numerical Studies of Two-Dimensional Vortex Motion by a System of Point Vortices," *Journal of the Physical Society of Japan*, Vol. 34, Jan. 1973, pp. 247-253.

[10] Bloom, A. M. and Jen, H., "Roll-Up of Aircraft Trialing Vortices Using Artificial Viscocity," *Journal of Aircraft*, Vol. 11, Nov. 1974, pp. 714-716.

Flow Breakdown for Wings in Ground Effect

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Nomenclature

A_{mom} = momentum area
AR = aspect ratio
b = wing span
C_L = lift coefficient
H = wing height above ground
L = wing lift
S = wing area
V = freestream velocity
W_d = downwash velocity
α_d = downwash angle measured from horizontal
ρ = density of air

Introduction

CLASSICAL theory predicts lift should increase as a wing approaches the ground. Wind tunnel tests show the opposite – lift deteriorates sharply at a certain height above the ground. The cause is flow breakdown. The theory of flow breakdown is presented here.

Theory

The momentum area for a wing is given in Ref. 1 as $\pi b^2/4$. From momentum requirements,

$$L = \rho A_{mom} V(2W_d) = C_L \tfrac{1}{2}\rho V^2 S \tag{1}$$

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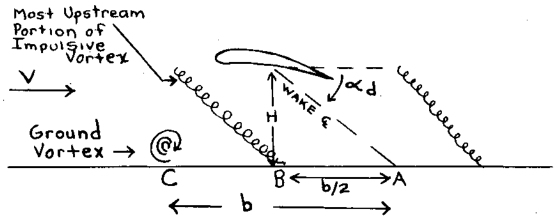

Fig. 1 Side view of impulsive vortex.

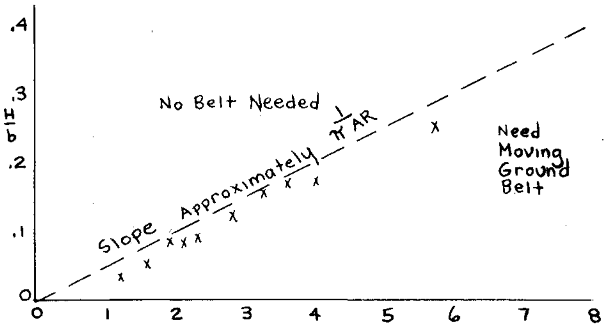

Fig. 2 Turner's graph.

Therefore,

$$A_{mom} = C_L VS/4W_d \tag{2}$$

The classical value of downwash at the wing is

$$W_d = (C_L/\pi AR)\, V \tag{3}$$

Combining Eqs. (2) and (3) gives

$$A_{mom} = \pi b^2/4 \tag{4}$$

The wing can be represented with an impulsive vortex of diameter b. The vortex is viewed from the side in Fig. 1.

From classical theory

$$\tan\alpha_d = W_d/V = C_L/\pi AR \tag{5}$$

When the upsteam portion of the vortex strikes the ground, it moves to position C, twice the normal distance from the wake centerline. This is proven in Ref. 2. This phenomenon can be understood by noting that were a ring vortex to strike a wall, the velocity through the center would effectively double because it would be encountering an "image" ring coming from the opposite direction. Therefore, for the circulation to remain constant, the diameter of the ring would double.

When the ground vortex is at C, upstream of the wing, it can exert downwash on the wing. This results in flow breakdown. Flow breakdown occurs when

$$\tan\alpha_D = H/b \tag{6}$$

But

$$\tan\alpha_d = W_d/V = C_L/\pi AR \tag{7}$$

Equating Eqs. (6) and (7) yields

$$H = bC_L/\pi AR \tag{8}$$

Equation (8) shows the height of the wing above the ground at which flow breakdown occurs. Flow breakdown theory correlates well with a number of references. South,[3] using H. Heyson's data,[1] shows for high lift devices that nonmoving